EXAMPLE 12.4

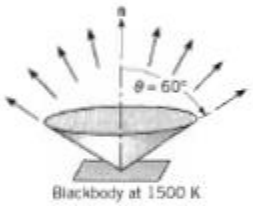
A surface emits as a blackbody at 1500 K. What is the rate per unit area (W/m^2) at which it emits radiation over all directions corresponding to $0° \leq \theta \leq 60°$ and over the wavelength interval $2\ \mu m \leq \lambda \leq 4\ \mu m$?

SOLUTION

Known: Temperature of a surface that emits as a blackbody.

Find: Rate of emission per unit area over all directions between $\theta = 0°$ and $60°$ and over all wavelengths between $\lambda = 2$ and $4\ \mu m$.

Schematic:



Assumptions: Surface emits as a blackbody.

Analysis: The desired emission may be inferred from Equation 12.10, with the limits of integration restricted as follows:

$$\Delta E = \int_2^4 \int_0^{2\pi} \int_0^{\pi/3} I_{\lambda,b} \cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda$$

or, since a blackbody emits diffusely,

$$\Delta E = \int_2^4 I_{\lambda,b} \left(\int_0^{2\pi} \int_0^{\pi/3} \cos\theta \sin\theta \, d\theta \, d\phi \right) d\lambda$$

$$\Delta E = \int_2^4 I_{\lambda,b} \left(2\pi \frac{\sin^2\theta}{2} \Big|_0^{\pi/3} \right) d\lambda = 0.75 \int_2^4 \pi I_{\lambda,b} \, d\lambda$$

Substituting from Equation 12.11 and multiplying and dividing by E_b, this result may be put in a form that allows for use of Table 12.1 in evaluating the spectral integration. In particular,

$$\Delta E = 0.75 E_b \int_2^4 \frac{E_{\lambda,b}}{E_b} d\lambda = 0.75 E_b [F_{(0\to 4)} - F_{(0\to 2)}]$$

where from Table 12.1

$$\lambda_1 T = 2\ \mu m \times 1500\ K = 3000\ \mu m \cdot K: \qquad F_{(0\to 2)} = 0.273$$

$$\lambda_2 T = 4\ \mu m \times 1500\ K = 6000\ \mu m \cdot K: \qquad F_{(0\to 4)} = 0.738$$

Hence

$$\Delta E = 0.75(0.738 - 0.273) E_b = 0.75(0.465) E_b$$

From Equation 12.26, it then follows that

$$\Delta E = 0.75(0.465) 5.67 \times 10^{-8}\ W/m^2 \cdot K^4 (1500\ K)^4 = 10^5\ W/m^2 \quad \lhd$$

Comments: The total, hemispherical emissive power is reduced by 25% and 53.5% due to the directional and spectral restrictions, respectively.